Production of the $\eta_b(nS)$ states [1]

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The rates for magnetic dipole (M1) transitions $\Upsilon(nS) \to \eta_b(n'S) + \gamma$, $n' \leq n$, are compared. The photon energies for allowed ($n' = n$) M1 transitions are very small, so hindered ($n' < n$) transitions could be more favorable for discovering the $\eta_b(1S, 2S)$. The question then arises whether $\Upsilon(2S)$ or $\Upsilon(3S)$ is a better source of $\eta_b(1S)$. Whereas one nonrelativistic model favors $\eta_b(1S)$ production from $\Upsilon(2S)$, this advantage is lost when relativistic corrections are taken into account, and is not common to all sets of wave functions even in the purely nonrelativistic limit. Thus, the prospects for discovering $\eta_b(1S)$ in $\Upsilon(3S)$ radiative decays could be comparable to those in $\Upsilon(2S)$ decays. We also discuss a suggestion for discovering the η_b via $\Upsilon(3S) \to h_b(^1P_1)\pi\pi$, followed by $h_b \to \eta_b\gamma$.

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The Υ $b\bar{b}$ resonances have a rich spectroscopy [1]. The spin-triplet S-wave levels $\Upsilon(nS)$ with $J^{PC} = 1^{--}$ are produced by virtual photons in hadronic or e^+e^- interactions, and then can undergo electric dipole (E1) transitions to the spin-triplet P-wave levels. However, to reach the spin-singlet S-wave levels $\eta_b(nS)$ from the easily-produced 1^{--} states, it is necessary to use either favored magnetic dipole (M1) transitions with very small photon energy, or hindered M1 transitions with change of principal quantum number. No spin-singlet $b\bar{b}$ levels have yet been seen. The mass splitting between the singlet and triplet states is a key test of the applicability of perturbative quantum chromodynamics (PQCD) to the $b\bar{b}$ system [2, 3] and is a useful check of lattice QCD results [4].

In this note we review some predictions for M1 transitions [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16] from the Υ (n^3S_1) levels to the η_b (n'^1S_0) states. The photon energies for allowed ($n' = n$) transitions are very small, so the hindered ($n' < n$) transitions could offer better prospects for discovering the spin-singlet states. The question then arises whether $\Upsilon(2S)$ or $\Upsilon(3S)$ is a better source of $\eta_b(1S)$. This question has taken on

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renewed interest because of plans of the CLEO Collaboration at the Cornell Electron Storage Ring (CESR) to increase their sample of data at the $\Upsilon(3S)$ and possibly the $\Upsilon(2S)$ resonance.

The answer is very dependent on good knowledge of relativistic corrections [8, 9, 10, 12, 14, 17]. In the absence of such corrections the only source of a non-zero overlap between nS and $1S$ wave functions in the hindered transitions is the variation with r of $j_0(kr/2)$, where k is the photon energy in the rest frame of the decaying particle and $j_0(x) \equiv (\sin x)/x$. In some (e.g., [5]) but not other (e.g, [10]) treatments, the matrix element of $j_0(kr/2)$ is much larger between $2S$ and $1S$ than between $3S$ and $1S$ wave functions. This hierarchy is largely obliterated when relativitistic corrections are taken into account. The most important of these appears to be the difference between ${}^1S_0$ and ${}^3S_1$ wave functions due to the hyperfine interaction. The attractive spin-spin interaction in the ${}^1S_0$ states causes the wave function to be drawn closer to the origin, leading to much less difference between the branching ratios for $\Upsilon(2S) \to \gamma\eta_b$ and $\Upsilon(3S) \to \gamma\eta_b$. More recently one group has pointed out a crucial role for exchange currents [14], which leads to very different conclusions from those obtained previously.

The rates for magnetic dipole transitions in quarkonium ($Q\bar{Q}$) bound states are given in the nonrelativistic approximation by [18, 19]

$$\Gamma({}^3S_1 \to {}^1 S_0 + \gamma) = \frac{4}{3}\alpha\frac{e_Q^2}{m_Q^2}I^2k^3 \quad , \tag{1}$$

where $\alpha = 1/137.036$ is the fine-structure constant, e_Q is the quark charge in units of $|e|$ ($-1/3$ for $Q = b$), and m_Q is the quark mass (which we shall take equal to 4.8 GeV/c^2). In all our discussions we shall assume a normal magnetic moment of the b quark. The overlap integral I is defined by

$$I = \langle f|j_0(kr/2)|i\rangle \quad . \tag{2}$$

We summarize in Table I some predictions for mass splittings between n^3S_1 and n^1S_0 $b\bar{b}$ levels and the corresponding branching ratios for $\Upsilon(nS) \to \eta_b(nS)+\gamma$ entailed by Eq. (1) assuming unit overlap integral, $I = 1$. We take the total widths of the $\Upsilon(nS)$ levels to be [20] $\Gamma_{\rm tot}[\Upsilon(1S, 2S, 3S)] = (52.5, 44, 26.3)$ keV. For the low-energy favored M1 transitions, the photon energies are nearly the same as the mass splittings. The wide variation in predicted hyperfine splittings leads to considerable uncertainty in predicted rates for these transitions.

For the higher-energy hindered M1 transitions, the expected photon energies $k = (M_i^2 - M_f^2)/(2M_i)$ are not so sensitive to hyperfine splittings. On the basis of present experimental values for the $\Upsilon(1S, 2S, 3S)$ masses [20] of (9460,10023,10355) MeV/c^2 and the hyperfine splittings predicted in Ref. [5], the masses for the η_b levels from are predicted to be $M(1S, 2S, 3S) = (9403, 9997, 10336)$ MeV/c^2, a representative set which we shall take in further calculations. We then compare two predictions for overlap integrals and branching ratios in Table II, *taking into account only the expectation value of the spherical Bessel function* $j_0(kr/2)$ *between initial and final*

Table I: Predictions for hyperfine splittings between n^3S_1 and n^1S_0 $b\bar{b}$ levels, and corresponding predicted branching ratios $\mathcal{B}$ for favored M1 transitions. Overlap integrals have been set equal to unity except in the second-to-last row.

Reference	$n = 1$		$n = 2$		$n = 3$	
	ΔM (MeV)	$\mathcal{B}$ (10^{-4})	ΔM (MeV)	$\mathcal{B}$ (10^{-4})	ΔM (MeV)	$\mathcal{B}$ (10^{-4})
MR83 [5, 6]	57	1.7	26	0.19	19	0.12
MB83 [7]	100	8.9	40	0.68	31	0.53
GOS84 [9] (a)	67	2.7	31	0.32	-3	0
GOS84 [9] (b)	78	4.2	37	0.54	27	0.35
GI85 [10] (c)	63	2.2	27	0.21	18	0.10
PTN86 [11]	35	0.38	19	0.07	15	0.06
FY99 [3]	53	1.3	(d)	(d)	(d)	(d)
ZSG91 [12]	48	0.99	28	0.23	(d)	(d)
EQ94 [13]	87	5.9	44	0.91	41	1.2
LNR99 [14]	79	4.4	44	0.91	35	0.76
LNR99 [14] (e)	79	3.6	44	0.63	35	0.50
UKQCD00 [15]	42	0.66	(d)	(d)	(d)	(d)
BSV01 [16]	36–55	0.4–1.5	(d)	(d)	(d)	(d)

[a] Scalar confining potential (favored by fit to P-waves).
[b] Vector confining potential.
[c] The splittings are based on masses rounded to 1 MeV, not the results rounded to 10 MeV as given in Ref. [10].
[d] Not quoted. [e] Results for fully relativistic calculation.

Table II: Predictions for overlap integrals and branching ratios in hindered M1 transitions between n^3S_1 and n'^1S_0 $b\bar{b}$ levels, *neglecting* relativistic corrections.

	$n = 2,\ n' = 1$ $k = 600$ MeV		$n = 3,\ n' = 1$ $k = 908$ MeV		$n = 3,\ n' = 2$ $k = 352$ MeV	
Reference	$\lvert I\rvert$	$\mathcal{B}$ (10^{-4})	$\lvert I\rvert$	$\mathcal{B}$ (10^{-4})	$\lvert I\rvert$	$\mathcal{B}$ (10^{-4})
JR83 [5]	0.02	0.92	< 0.002	< 0.06	~ 0.005	0.01–0.04
GI85 [22]	0.017	0.67	0.0070	0.65	0.018	0.25
ZSG91 [12] (a)	0.069	11.0	(b)	(b)	(b)	(b)
ZSG91 [12] (c)	0.022	1.11	(b)	(b)	(b)	(b)
LNR99 [14]	(b)	0.78	(b)	1.1	(b)	0.54

[a] Scalar-vector exchange potential of Ref. [21]. [b] Not quoted.
[c] Scalar exchange potential of Ref. [21].

Table III: Predictions for overlap integrals and branching ratios in hindered M1 transitions between n^3S_1 and n'^1S_0 $b\bar{b}$ levels, taking into account relativistic corrections.

	$n=2,\ n'=1$		$n=3,\ n'=1$		$n=3,\ n'=2$	
	$k=600$ MeV		$k=908$ MeV		$k=352$ MeV	
Reference	$\lvert I\rvert$	$\mathcal{B}$ (10^{-4})	$\lvert I\rvert$	$\mathcal{B}$ (10^{-4})	$\lvert I\rvert$	$\mathcal{B}$ (10^{-4})
ZB83 [8]	0.080	15	0.041	22	0.095	7.0
GOS84 [9] (a)	(b)	7.9	(b)	(b)	(b)	(b)
GOS84 [9] (c)	(b)	5.4	(b)	(b)	(b)	(b)
GI85 [10] (d)	0.057	7.4	0.029	11	0.054	2.2
GI85 [22] (e)	0.081	13	0.043	25	0.078	4.7
ZSG91 [12] (f)	0.025	1.4	(b)	(b)	(b)	(b)
ZSG91 [12] (g)	0.001	~ 0	(b)	(b)	(b)	(b)
LNR99 [14] (h)	(b)	0.46	(b)	1.4	(b)	0.13
LNR99 [14] (i)	(b)	0.05	(b)	0.05	(b)	0.40

[a] Scalar confining potential. [b] Not quoted.
[c] Vector confining potential.
[d] Based on quoted transition moments.
[e] Based on matrix elements between $^3S_1$ and $^1S_0$ wave functions.
[f] Scalar-vector confining potential of Ref. [21].
[g] Scalar confining potential of Ref. [21].
[h] Without exchange current. [i] With exchange current.

states. While the overlaps for the $n = 2,\ n' = 1$ transition are similar, the other overlaps differ significantly, suggesting that they may be sensitive to small details of the potential (as in the case of two different potentials [21] utilized by Ref. [12]).

When relativistic corrections are taken into account, the results are as shown in Table III. Several calculations [8, 10, 22] predict large branching ratios for all sets of hindered transitions. There appears to be no particular advantage in searching for the $\eta_b(1S)$ state at the $\Upsilon(2S)$; the branching ratio from the $\Upsilon(3S)$ is predicted to be slightly larger, partly compensating for the lower production cross section of the $\Upsilon(3S)$. (The cross sections for $e^+e^- \to \Upsilon(2S)$ and $e^+e^- \to \Upsilon(3S)$ are measured to be about 6.6 and 4 nb, respectively [23, 24, 25], for final states other than lepton pairs.)

The calculation of Ref. [22] is based entirely on the distortion of the spin-singlet wave functions by the strong hyperfine attraction. The spin-independent potential consists of a short distance Coulomb-type Lorentz vector and a long distance linear Lorentz scalar interaction. The hyperfine interaction is included directly in the Hamiltonian by smearing the relative coordinate over distances of the order of the inverse quark masses which has the consequence of taming the singularities present in the Breit-Fermi interaction. The resulting wave functions are compared with the corresponding spin-triplet wave functions in Fig. 1. The stronger peaking near the origin of the singlet wave functions is clearly visible.

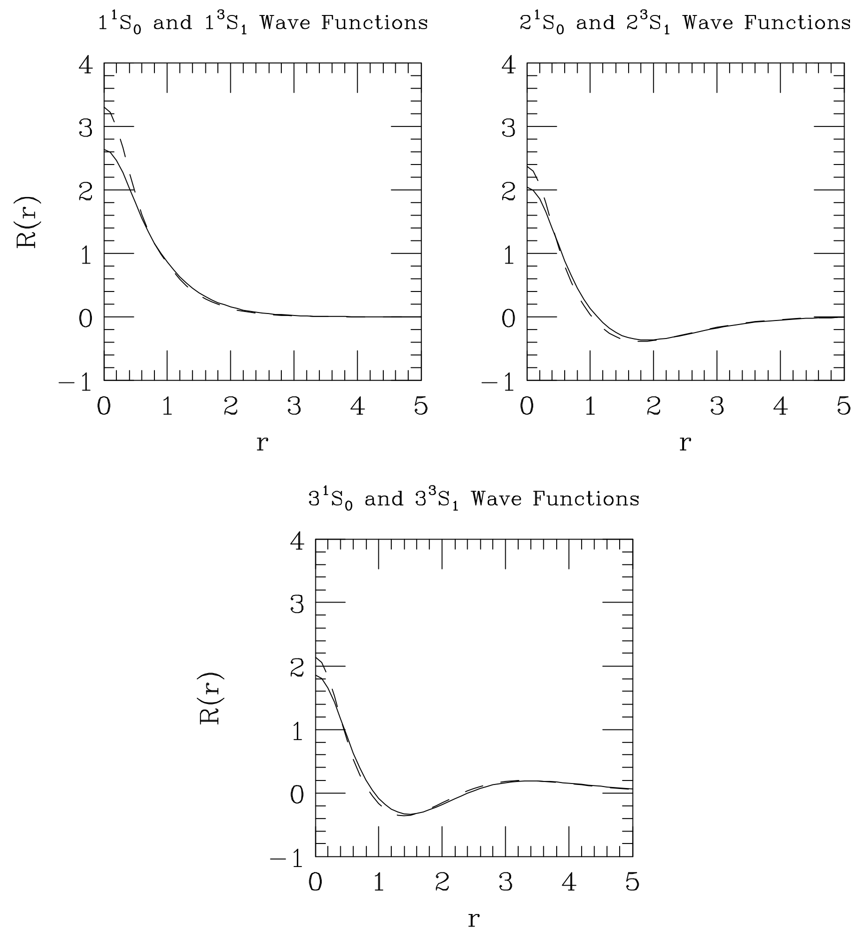

Figure 1: Comparison of spin-singlet (dashed) and spin-triplet (solid) wave functions for nS $b\bar{b}$ levels.

The best of present data [23, 24, 25] may not be adequate to confront such predictions, and no published limits are reported at present. For example, at the $\Upsilon(2S)$, the transition to the $\chi_{b0}(1P)$ and a 162 MeV photon corresponds to a signal of 8637 ± 1274 events in CLEO data [23], for a branching ratio of $(3.4\pm0.5\pm0.6)\%$. Although a considerable extrapolation is needed to anticipate the signal of a 600 MeV photon (since the spectrum for that energy is not published), it is likely that such a photon emitted with a branching ratio of 10^{-3} would be lost in the combinatorial background associated with neutral pions. Similarly, in cascade decays from the $\Upsilon(3S)$ to the $\chi'_{bJ}(2P)$ states followed by $\chi'_{bJ}(2P) \to \Upsilon(1S)\gamma$, the 770 MeV photon corresponds to a signal of 1994 ± 150 events in CUSB data [25], for a branching ratio of $(2.0 \pm 0.2 \pm 0.2)\%$. This is to be compared with the sought-for signal of a $\sim$ 900 MeV photon emitted with a branching ratio of about 2×10^{-3}. The corresponding spectrum for the CLEO $\Upsilon(3S)$ data, based on a smaller sample, shows similar features [26].

We conclude with some remarks on the production of the $\eta_b(1S)$ level through the decay $\Upsilon(3S) \to h_b(1^1P_1)\pi\pi$, which is predicted by Kuang and Yan [27] to have a branching ratio of about 0.1–1%. [Voloshin [28] finds a much smaller value for

this quantity, less than 10^{-4}, and suggests observing instead the isospin-violating transition $\Upsilon(3S) \to h_b(1^1P_1)\pi^0$, for which he predicts a branching ratio of 10^{-3}.] The subsequent electric dipole decay $h_b \to \eta_b(1S) + \gamma$ is predicted [27] to have a branching ratio approaching 50%. The mass of h_b is expected to be not far from the center-of-gravity of the 1^3P_J χ_b levels, or about 9.90 GeV/c^2, so the photon should have an energy of about 485 MeV in the h_b rest frame. The CUSB Collaboration has searched for the $h_b(1^1P_1)\pi\pi$ signature and is able to place an upper limit at 90% c.l. [25] of $< 0.45\%$ on the combined branching ratio for an $\Upsilon(1S)$–$\eta_b(1S)$ splitting ranging between 50 and 110 MeV. The CLEO Collaboration [29] places an inclusive upper limit of $\mathcal{B}[\Upsilon(3S) \to \pi^+\pi^- h_b] < 0.18\%$ at 90% c.l. for $M_{h_b} = 9.900$ GeV/c^2 and 90% c.l. upper limits in the range of 0.1% for the cascade $\mathcal{B}[\Upsilon(3S) \to \pi^+\pi^- h_b \to \pi^+\pi^-\eta_b\gamma]$, with $M_{h_b} = 9.900 \pm 0.003$ GeV/c^2 and a photon energy between 434 and 466 MeV.

It appears that the richness of transitions to $\eta_b(nS)$ levels available in decays of the $\Upsilon(3S)$ make it a promising initial candidate for enhanced searches for the elusive spin-singlet levels.

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